Fuzzy modeling and control of fed-batch fermentation

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Abstract
The goal of this paper is to propose an alternative to classical control methods of a fed-batch fermentation. Fuzzy modeling and control theory is used for that purpose. The fuzzy supervisor based on human experience and experimental data, is designed to compute the set values for a PI controller which controls the feeding dilute flow to the fermentator. The results are compared with those obtained by the classical optimal control.
Keywords
fuzzy modeling, fuzzy control, fed-batch fermentation

1. Introduction

Basically, the control task of fed-batch fermentation lies in the determination of the proper feed rate F of a substrate s that facilitates both the growth of the cell concentration x and the concentration of the desired product p within the volume V of the fermentator during the fermentation time. According to Viestur (1992), if an exact mathematical model exists, the control task can be solved using the optimal control theory. The analytical model of fed-batch fermentation has the following general form:

$$\begin{cases} \frac{dxV}{dt} = \mu xV \\ \frac{dsV}{dt} = -q_s xV + s_n F \\ \frac{dpV}{dt} = q_p xV \\ \frac{dV}{dt} = F \end{cases}, \qquad (1)$$

where $\mu = f(x, s, p)$; $q_p = f(x, s, p)$; $q_s = f(x, s, p)$, and s_n is the concentration of substrate in the feeding dilution.

Physical limitations are set for $V(T) = V_k$ (where T is the duration of the process) and F: $0 \leq F_{\min} < F < F_{\max}$. The control task involves the determination of such feed rate function that maximises the cost function $J(F)$=f(x_k, s_k, p_k, V_k, T), where x_k, s_k and p_k are the values of x, s and p at the moment T .

To solve this task, the initial values x_0, s_0, p_0 and V_0 of x, s, p and V must be given. The solution process is described in detail in (Viestur 1992).

2. Process parameters

Determining the mathematical model can often be a very complicated and time-consuming task. Moreover, using non-accurate model in the calculations of the optimal feed rate may lead to undesirable results. To demonstrate how appropriate control profiles can be found without the need for exact mathematical model of the process, we consider penicillin fed-batch fermentation as a sample fermentation process.

According to Viestur (1992), penicillin fed-batch fermentation is defined by Eqs. (1) with the following parameter values:

$$\mu = \frac{0.11s}{0.0006x + s},\ q_s = \frac{\mu}{0.47} + \frac{q_p}{1.2} + 0.029,\ q_p = \frac{0.004s}{0.0001 + s + 10s^2} - 0.01p$$

x_0 = 10.5 g; $s_0 = 10^{-6}$ g; p_0 = 0.0 g; V_0 = 7 l; V_k = 10 l; $F_{\max}$ = 100 ml/h; s_n = 100 g/l

The cost function for the optimal control task is defined as $J = p(T)V(T)$ and the duration of the fermentation process is not fixed.

The calculation of optimal feed rate itself is outside the scope of the present paper, but optimal feed rate (taken directly from (Viestur 1992)) and the corresponding results are used for comparison.

3. Fuzzy model

Instead of the mathematical model, the control information will be extracted from the fuzzy model of the process. Such model is based on input-output readings collected from the experiments conducted with the fermentation system. The power of the approach lies in the fact that the fuzzy model of the system, which describes the behaviour of the system in linguistic terms, is easily comprehensible to human beings, and the process controller can be obtained by the logical analysis of the rules of the model.

From many fuzzy modeling techniques available, the fuzzy template modeling via the belief structures proposed by Yager and Filev, is used here. The use of this method results in the formation of the Mamdani-type model of the process, while most of the fuzzy modeling techniques do not possess this feature. The original algorithm (for SISO systems) presented in (Yager & Filev 1994) was expanded to make it applicable in our case.

The method has a serious drawback - the structure of the model (fuzzy partitions of the input-output variables, structure of the rules) must be predefined. The procedure (identification of the structure) itself is not clearly defined and is therefore more an art than science.

The structure of the model defines the initial rulebase of the model that consists of all the possible combinations between the predefined fuzzy sets of input-output variables. The next step of modeling is tuning the individual weights of the rules according the input-output data. The weight of a rule expresses the relevance of this particular rule in the model.

To find the basic relationships between the fermentation components, we try to identify the model, which has the following structure:

IF x is A AND s is B THEN dx is C AND dp is D, (2)

where dx and dp are the growth rates of the biomass and product, respectively, and A, B, C and D are fuzzy sets or linguistic variables defined over input and output universes of discourse such as “low”, “average”, “high”, etc. Selection of the amount and parameters of such linguistic variables is the most crucial task in this approach. Model (2) is expected to give us all the information we need to design the control system.

4. Input-output data and fuzzy partitioning

Choosing the input-output data for modeling is a complicated task. Data used for modeling should adequately reflect the behaviour of the system. Little data increases the risk that important information would be missed. At the same time, huge amounts of information, slow down the modeling, complicate the design of the control system and consequently, increase the overall design cost.

We have limited ourselves to constant feed rates when making experiments to produce the input-output data for modeling. The amount of possible constant feed rates that fulfils the condition (3) is infinite.

$$F = \frac{V - V_0}{t}. \tag{3}$$

However, feed rates cannot exceed physical limitations (F_{max}), and conducting too many experiments is unreasonable. In particular, too high or too low feed rates are outside consideration.

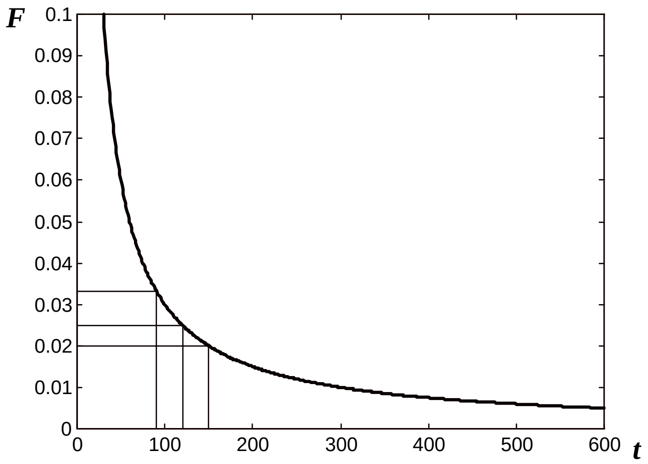

Fig. 1. Possible constant feed rates.

In the present work, the experimental data was collected from the simulations made with three different constant feed rates F=0.02, 0.025, 0.033, as shown in Fig. 1.

The fuzzy partitioning of the model variables into fuzzy sets is an extremely ill-defined process. The fuzzy sets should be selected so that the resulting model is adequate, and very often human experience about the process is used in conjunction with the observed data. Partitions here are based purely on our assumptions about the nature of the process, made on the basis of input-output readings. Perhaps the best example of our partitioning strategy is the partition of substrate concentration s shown in Figs. 2-4.

At the beginning of the process (Fig. 2) s grows very fast and reaches high values quickly, then falls to an extremely low level (Fig. 3). Although the latter is numerically very little as compared to the maximum values, it must not be ignored and should be represented by respective fuzzy sets (Fig. 5). Other variables (x, dp, dx) have been partitioned basically in the same mood.

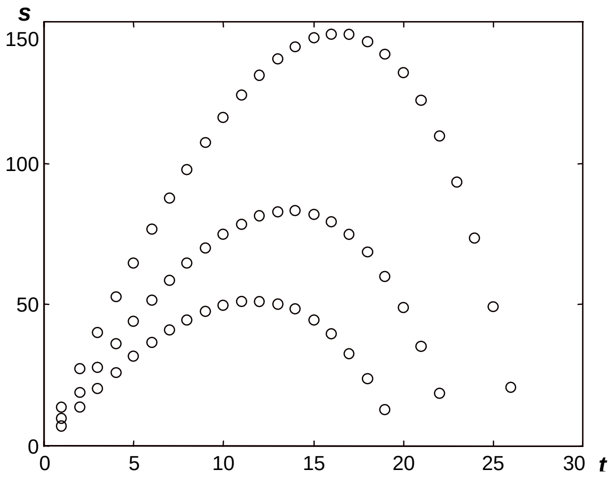

Fig. 2. Substrate concentration in the beginning of the process.

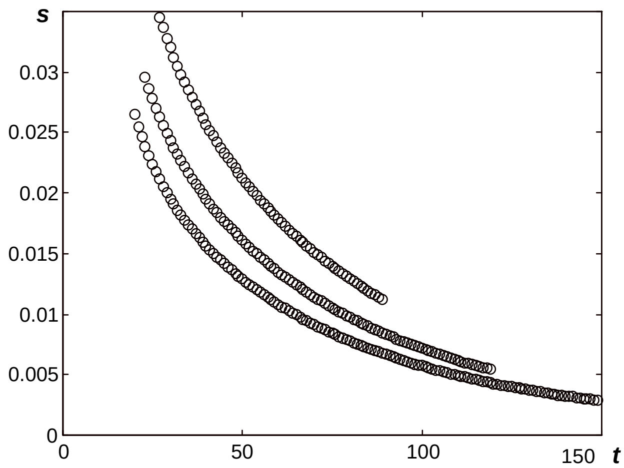

Fig. 3. s in the second part of the process.

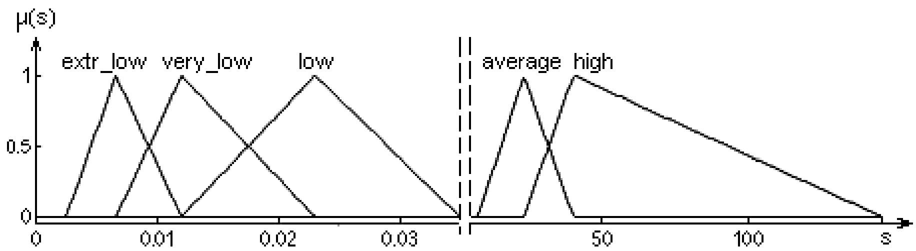

Fig. 4. Fuzzy sets of s.

5. Designing the supervisor

The model obtained (2) contains the information how the two outputs - dp and dx are affected by the different qualitative measures of x and s. As a result of analysing this information, the control strategy can be derived.

To achieve large quantities of p by the end of the process, the control strategy, first of all, must guarantee high product growth rate during the process. The rules in (2) give us the concentrations of s and x that lead to such growth rates. It turns out that the highest possible growth rates of p occur while the biomass concentration is sufficiently "high" in the fermentator. Therefore, the control actions must lead first to the high biomass concentrations and then maintain this level until the end of the process. The fuzzy sets A of x themselves act as the identifiers of the different stages of the process. Matching all the biomass concentrations with appropriate substrate concentrations in accordance with the fuzzy values of dp and dx results in the formation of the fuzzy supervisor, where every A is paired with appropriate B (4).

$$\text{IF } x \text{ is } A \text{ THEN } s \text{ is } B \qquad (4)$$

Thus, a fuzzy supervisor is a set of rules each of which specifies which B of s must be maintained in the fermentator during the stage A.

Usually such analysis leads to more than one version of the supervisor.

6. Control system

Fuzzy supervisor will be embedded in the control system, where it calculates the set values for the conventional PI controller that controls the feeding rate (Fig. 5).

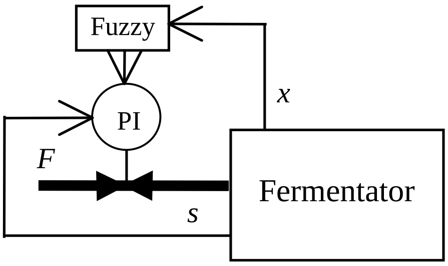

Fig. 5. Control of penicillin fermentation using fuzzy supervisor.

7. Results

The discrete time fermentation system based on Eqs. (1), the control system, and modeling program were implemented in MATLAB 4.3.

The structure of the fuzzy model of the system was defined by the fuzzy sets of the variables presented in Table 1.

variable	**membership functions**				
x	low	below average	average	high	
s	extremely low	very low	low	average	high
dp	zero	low	average	high	
dx	low	below average	average	high	

Table 1. Membership functions of model variables

The initial model consisted of 320 elementary rules. After tuning the model and removing all the rules with zero or very small weights, this number was reduced to 53. The analysis of these rules gave three substantially different versions of the supervisor (Table 2). Further comparison of these scenarios is possible only making real fermentation or process simulations. Figures 6 and 7 show the results of using the derived supervisors.

x	**version 1**	**version 2**	**version 3**
low	high	high	high
below average	high or average	high	small
average	high or average	extremely small	extremely small
high	extremely small	extremely small	extremely small

Table 2. Fuzzy supervisors

During the fermentation, two major phases - biomass growth phase and production phase - can be distinguished, and scenarios 1-3 differ essentialy by the moment the transition takes place.

Following scenario 1, the latest transition is the case, biomass can achieve the highest concentration possible by the beginning of the production phase, and therefore the growth rate of the product in the production phase is also the highest. However, very large quantity of the product cannot be achieved, because the fermentator fills up too soon. On the other hand, scenarios 2 and 3 offer good final product concentrations, comparable to the one obtainable with optimal control but require more time.

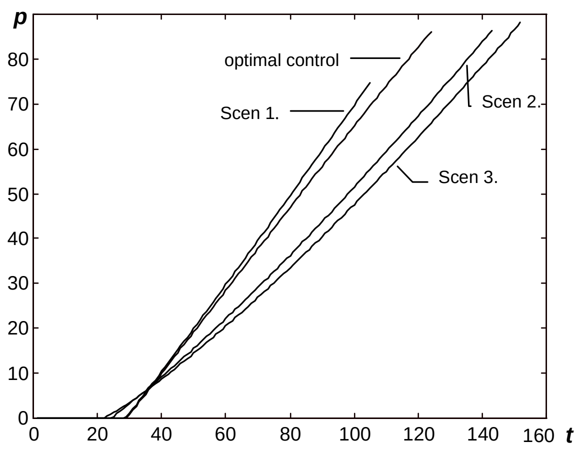

Fig. 6. Product concentrations during the fermentation.

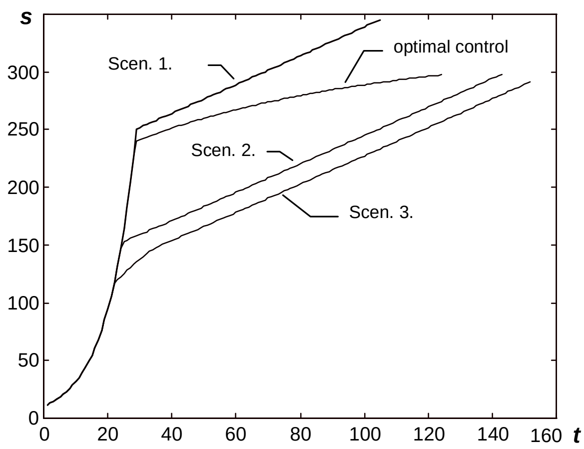

Fig. 7. Biomass concentrations during the fermentation.

One can notice that in the case of scenario 1 the fermentation productivity ($V(T)p(T)/T$) is even higher than that of optimal control. In real life, the advantage of a scenario is determined also by many other circumstances (substrate cost, fermentation preparation cost, time delay between the individual fermentations). The most suitable scenario can be finally chosen taking into account all such considerations.

8. Conclusions

Fuzzy modeling theory provides tools for handling the systems that are too complex or ill-defined to apply the conventional modeling and control methods. Our example was chosen to demonstrate that even without the mathematical model of the process, the control algorithms can be derived that produce desirable results. Fuzzy model of the process that is based on input-output data gives us the knowledge necessary to synthesise the control system.

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