TRANSPARENCY ANALYSIS OF FIRST-ORDER TAKAGI-SUGENO SYSTEMS

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Abstract

The primary goal of this paper is to investigate transparency properties of Takagi-Sugeno (TS) systems. TS systems, distinguished by half-linguistic/half-functional rules are exploited in numerous applications today and deserve special attention from transparency viewpoint. The fact that TS systems can be interpreted in terms of local linear models is generally acknowledged. Few authors, however, have been interested with the question if this interpretation can be considered accurate (i.e. if the system is transparent). In this paper, the problem is taken into consideration and transparency measure for 1st order TS systems is introduced. Included are examples of TS modeling demonstrating that transparency and adaptability of TS systems are of somewhat exclusive character.

1. Introduction

With fuzzy systems a new term - "linguistic interpretation" - has been added into engineer’s vocabulary. Standard (Mamdani) fuzzy systems enable one to process information in qualitative terms and natural language thus giving a system engineer an additional set of tools for synthesis and analysis of engineering problems [Passino 1998]. The linguistic interpretation is valid and can be exploited only if certain conditions concerning the membership functions of the system are satisfied as shown in [Riid 2000a]. We call the systems satisfying these conditions transparent. The modeling algorithms that have no built-in transparency protection tend to yield non-transparent fuzzy models with confusing and even meaningless rules - by that fuzzy logic is reduced to a black-box technique like neural networks and the basic advantage that fuzzy systems have over neural networks is lost.

With TS systems the matters are more complicated as due to the differences in system architecture the transparency constraints cannot be defined explicitly. In this paper we investigate the relationship between adaptability and transparency in TS systems in order to develop some practical guidelines for obtaining transparent systems.

2. TS systems and transparency

Consider a multi-input/single-input first-order TS fuzzy system (1) consisting of R rules, defuzzified by the fuzzy-mean method, where A_{ir} denote the linguistic labels of the i^{th} input variable, associated with the r^{th} rule, having one-to-one correspondence with normal convex spline-based [Bikdash 1998] membership functions (MFs) μ_{ir}; a_{ir} denote the consequent parameters of the r^{th} rule, x_i denotes the numerical value of the i^{th} input variable and ∩, ∪ are the operators called t-norm and t-conorm, respectively.

$$\begin{cases} \text{IF } x_1 \text{ is } A_{1r} \text{ AND} \ldots \text{AND } x_i \text{ is } A_{ir} \text{ AND} \ldots \\ \text{AND } x_N \text{ is } A_{Nr} \text{ THEN } y_r = a_{0r} + a_{1r}x_1 + \ldots \text{a}_{Nr} x_N, \\ \\ y = \dfrac{\bigcup_{r=1}^{R} \left(\left(\bigcap_{i=1}^{N} \mu_{ir}(x_i) \right) \cap (a_{0r} + \ldots + a_{ir}x_i + \ldots \text{a}_{Nr} x_N) \right)}{\bigcap_{i=1}^{N} \mu_{ir}(x_i)}, \end{cases} \quad (1)$$

First-order TS systems are interpreted in terms of local linear models [Babuška 1997] therefore TS system can be considered transparent if its global output y can be derived directly on the basis of its local outputs y_r. The influence of input partition to the transparency of TS systems is noted in a number of works [Babuška 1997], [Oliveira 1999], [Riid 2000b]. Although input transparency condition (2), given in [Riid 2000c] for standard fuzzy systems

$$\forall x_i \in X_i : 0 < \sum_{s=1}^{S_i} \mu_i^s(x_i) \leq 1, \quad (2)$$

where μ_i^s denotes the s^{th} MF of the i^{th} input variable, $X_i = [X_i^{\min}, X_i^{\max}]$, is relevant, it is, however, not sufficient for making TS systems transparent. Consider a SISO first-order TS system depicted in Fig. 1.

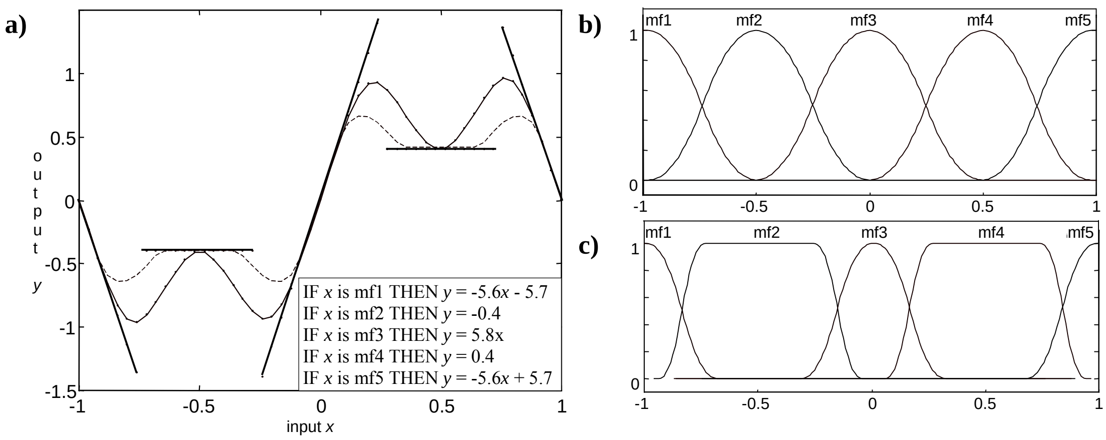

Fig. 1. Input-output relation of the SISO TS system (a), original input partition of the system (b) and input MFs with modified cores (c).

If the cores of input MFs *mf2* and *mf4* are enlarged (Fig.1c), global output (dashed line) deviation from local outputs is reduced in Fig 1a, but for substantial part global output is the result of interpolation of individual rules. This interpolation is the reason why in several regions interpretation of the model behavior based purely on local outputs would lead to erroneous conclusions.

3. Interpolation issues

Babuška has analyzed the interpolation issues of TS systems [Babuška 1997] and distinguishes S-type and V-type interpolation, by his definition the rules of the system in Fig.

1 belong to S-type. Basic conclusions made by him about different interpolation types are summarized into Table 1.

Table 1.

Interpolation type	Interpolation properties	Application area
S-type interpolation	intuitively expected results	Stepwise and possibly discontinuous function approximation
V-type interpolation	some undesirable properties	Continuous, smooth function approximation

According to Table 1 neither of the interpolation types has clear advantage over another. In [Babuška 1997], however, preference seems to be given to V-type and weighted-mean defuzzification algorithm is replaced by another functional, i.e. *smoothing maximum*. This replacement can be considered a deviation from the "classic" TS inference algorithm, and is not accepted in the current paper.

The distinction between V and S-type rules is given in general terms. According to that, a pair of affine rules (R_i, R_j) is of the V-type if and only if

$$\Omega_{ij} \cap S_{ij} \neq 0 \text{ and } \Omega_{ij} \cap (C_i \cup C_j) \text{ ,} \tag{3}$$

where Ω_{ij} denotes the intersection of the consequents of R_i and R_j projected on $\boldsymbol{x}$, the vector of input variables; S_{ij} denotes the support of the intersection of affine membership functions associated with these rules and C_i, C_j denote the cores of the respective affine rules. With clear preference given to V-type interpolation, (3) should be maintained for all rules throughout the training process.

We observe how to apply (3) to TS systems with single input and then with two inputs to give an illustration of the complexity of the problem.

First, let us consider a single input/single output TS system. Assuming that we are employing four-parameter input MFs ($\mu(\alpha)=\mu(\delta)=0, \mu(\beta)=\mu(\gamma)=1$, $\alpha \le \beta \le \gamma \le \delta$), (3) is satisfied if for two affine rules R_i and R_{i+1} the following holds.

$$\gamma_i < \frac{a_{0i} - a_{0,i+1}}{a_{1,i+1} - a_{1i}} < \beta_{i+1} \text{ , } i = 1 \ldots R - 1 \tag{4}$$

For the system with two inputs each rule R_{ij} has four affine rules: $R_{i-1,j}$, $R_{i+1,j}$, $R_{i,j-1}$, $R_{i,j+1}$ (Fig. 2) except for those R_{ij} that are positioned at the extremes of the domain; for the latter special conditions apply.

The projection of the intersection of R_{ij} and any of its affine rules onto the input space results in a line. The coordinates of intersection points of these lines can be found from the following equation systems.

$$p_{ij}^1 : \begin{cases} y_{ij} = y_{i+1,j} \\ y_{ij} = y_{i,j+1} \end{cases} \quad p_{ij}^2 : \begin{cases} y_{ij} = y_{i+1,j} \\ y_{ij} = y_{i,j-1} \end{cases} \quad p_{ij}^3 : \begin{cases} y_{ij} = y_{i-1,j} \\ y_{ij} = y_{i,j-1} \end{cases} \quad p_{ij}^4 : \begin{cases} y_{ij} = y_{i-1,j} \\ y_{ij} = y_{i,j+1} \end{cases} \tag{5}$$

Thus $(x_1^{p_{ij}^1}, x_2^{p_{ij}^1})$, the coordinates of p_{ij}^1 can be found from the respective equation system (6)

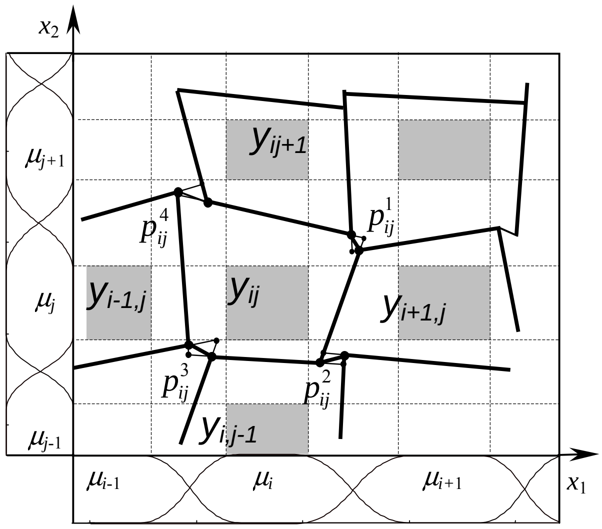

Fig. 2. The projection of input-output relation onto the input space of two input/single output TS system.

$$x_2^{p_{ij}^1} = \frac{(a_{0i+1j} - a_{0ij})(a_{1ij} - a_{1ij+1}) - (a_{1ij} - a_{1i+1j})(a_{0ij+1} - a_{0ij})}{(a_{1ij} - a_{1i+1j})(a_{2ij+1} - a_{2ij}) + (a_{1ij} - a_{1ij+1})(a_{2ij} - a_{2i+1j})} \tag{6}$$

$$x_1^{p_{ij}^1} = \frac{a_{0ij+1} - a_{0ij} - x_2^{p_{ij}^1}(a_{2ij} - a_{2ij+1})}{a_{1ij} - a_{1ij+1}}$$

Now, (3) translates into the following form

$$\begin{cases} \gamma_i < x_1^{p_{ij}^1} < \beta_{i+1} \\ \gamma_j < x_2^{p_{ij}^1} < \beta_{j+1} \end{cases} \qquad \begin{cases} \gamma_{i-1} < x_1^{p_{ij}^2} < \beta_i \\ \gamma_{j-1} < x_2^{p_{ij}^2} < \beta_j \end{cases} \tag{7}$$

The procedure is to be repeated for all N_p points to ensure the V-interpolation for all rules

$$N_p = 2^N \left(\prod_{i=1}^{N} S_i - 1 \right), \tag{8}$$

where S_i is the number of MFs per i^{th} input variable and N is the number of inputs.

With three or more inputs the computation of (3) is getting even more complicated and even if the decision is made in favor of V-interpolation, there are no effective means for preserving that type of interpolation during the training of TS systems.

4. Modeling experiments

In order to approach the problem from the practical point of view we have prepared a set of data generated by

$$\hat{y} = 0.6\sin(\pi x) + 0.3\sin(3\pi x) + 0.1\sin(5\pi x), \tag{9}$$

where x is discretized in [-1,1] with the step 0.05 for training 0^{th} and 1^{st} order TS systems using ANFIS [Jang 1993].

Unlike binary-valued transparency of Mamdani and 0^{th} order TS systems, 1^{st} order TS systems are characterized by a varying degree of transparency. The following measure based on the deviation of the global output from local outputs of the system is proposed:

$$\varepsilon_{tr} = \sqrt{\frac{\sum_{k=1}^{N} (\hat{y}(k) - \underset{\max \tau_r(k)}{y_r}(k))^2}{N}}, \qquad (10)$$

where $\underset{\max \tau_r(k)}{y_r}$ denotes the local output of the r^{th} rule having the maximum firing strength for the k^{th} input-output pair.

To obtain material for comparison, several models were used, with the different number of MFs. The modeling results including modeling number of input MFs (S), root-mean-square modeling errors (*RMSE*), transparency measure (ε_{tr}) and number of training epochs (N_e) are presented in Table 2.

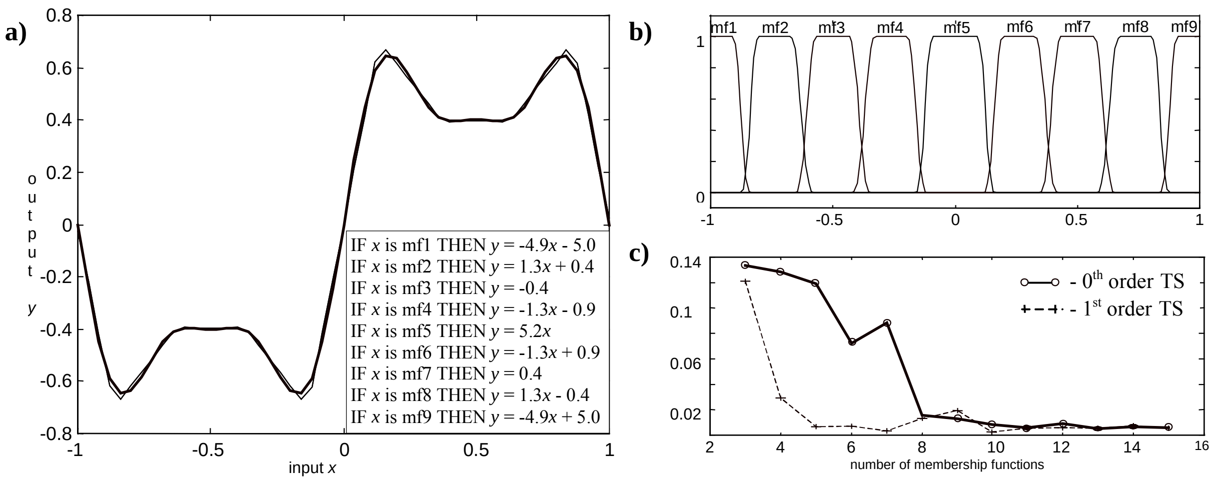

Fig. 3. Modeling results: model output (thin line) plotted against training data (bold line) (a), input MFs of the model (b), correlation between the modeling error and the number of MFs (c).

First, not paying attention to transparency measure, it is obvious that from adaptability point of view 1^{st} order TS systems are much more flexible than 0^{th} order systems (Fig 3c). Sufficiently small approximation error is achieved only with 5 input MFs, as with 0^{th} order TS systems at least 8 MFs are required. Inspection of τ_r, however, reveals very few cases where the interpretation in terms of local linear models can be considered accurate (7,12,13). The best result (result no. 7 in Table 2) is depicted in Figs. 3a (curve) and 3b (input MFs). Very distinct MFs can be noticed with what the role of interpolation is strongly reduced. With few MFs (and poor transparency measure), on the other hand, interpolation plays important role in overall output. The relationship is obvious but "large enough" number of input MFs unfortunately does not automatically make the system transparent. To achieve both goals (transparency and low approximation error) simultaneously, an optimal number of MFs must be specified that, of course, varies from case to case and cannot be determined beforehand. Additional experiments with MISO systems have shown these results being basically valid with MISO systems.

Table 2. Modeling results.

		0th order TS systems		1st order TS systems		
exp.no	S	*RMSE*	N_e	*RMSE*	N_e	ε_{tr}
1	3	0.13363	200	0.1210	100	0.0798
2	4	0.12844	60	0.0290	60	0.1023
3	5	0.11917	100	0.0065	60	0.0850
4	6	0.07217	100	0.0068	70	0.1191
5	7	0.08840	80	0.0034	100	0.0816
6	8	0.01576	120	0.0133	60	0.0942
7	9	0.01263	100	0.0193	20	0.0144
8	10	0.00828	80	0.0023	60	0.0639
9	11	0.00561	40	0.0052	60	0.0466
10	12	0.00909	100	0.0058	60	0.0455
11	13	0.00478	25	0.0054	40	0.0223
12	14	0.00655	25	0.0075	25	0.0194
13	15	0.00575	40	0.0054	40	0.0119

5. Conclusions

In the light of the results of the present work it is quite simple to build a perfectly transparent TS system - box-shaped MFs that reduce the interpolation factor to zero will do the job. Two problems arise - a) can such system considered a fuzzy systemat all; b) the excellent approximation properties of TS systems would be largely lost. Otherwise there is no good guarantee of transparency, moreover, improved transparency would basically mean deteriorated adaptability and vice versa. Transparency measure (10) incorporated into approximation criterion may influence the training process toward increased transparency. The crucial question in this procedure is how to determine the appropriate number of input MFs that as could be easily imagined, is very difficult in case of MISO systems and what, when suboptimal, results in loss of transparency.

6. References

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